

# Phase Gradient Rate-Constrained Minimum Cost Flow Method for Accurate Phase Unwrapping in Large Gradient Deformation Fields (Project ID: 95348)

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## ABSTRACT

Phase unwrapping (PU) is a critical step in InSAR processing, directly impacting the accuracy of ground deformation measurements. Traditional PU methods—such as Minimum Cost Flow (MCF), Snaphu, and deep learning models like CNN-SQD-LSTM and PUNet—often fail in regions with large deformation gradients, where the assumption of phase continuity is violated. To address this challenge, we propose the **Phase Gradient Rate Constrained MCF (PGR-MCF) method**. It introduces Phase Gradient Rate (PGR), derived by stacking multiple differential interferograms, as a constraint to guide unwrapping paths in the MCF framework. High-coherence pixels are connected using Delaunay triangulation, and arcs with high PGR values are assigned zero weight to avoid unreliable regions. This strategy steers the unwrapping path from stable low-gradient zones into the deformation field. Simulations demonstrate that PGR-MCF achieves 99.5% accuracy in high-gradient scenarios. In the Guobu landslide case, it enhances spatial continuity, reveals internal deformation fronts missed by other methods, and reduces the RMSE between InSAR and GNSS time series by over 65%. PGR-MCF performs robustly on interferograms with long temporal baselines, increasing the number of usable observations for time series analysis. Fully data-driven and independent of external models, the method adapts well to diverse InSAR datasets and terrain conditions. While limitations may occur when the entire boundary exhibits high gradients, PGR-MCF consistently delivers improved accuracy and interpretability in most real-world settings. It is especially suitable for large-scale deformation monitoring and rapid hazard assessment.

## INTRODUCTION

InSAR estimates surface deformation by analyzing the phase difference between SAR images acquired at different times. Since its development, InSAR has been widely applied in terrain mapping and deformation monitoring. However, the accuracy of deformation results strongly depends on phase unwrapping (PU), a crucial processing step that reconstructs absolute phase values from wrapped observations. Conventional PU methods—such as MCF, Snaphu, and optimization-based approaches—generally assume phase continuity between neighboring pixels. This assumption often breaks down in regions with low coherence or large deformation gradients, resulting in significant unwrapping errors.

Recent deep learning-based PU models, such as CNN-SQD-LSTM and PUNet, show improved robustness under complex conditions. However, their performance relies heavily on the availability and representativeness of training data and may degrade when applied to new deformation scenarios. Moreover, these models lack physical interpretability and are not yet widely adopted in operational InSAR applications.

To overcome these limitations, we propose a novel strategy that incorporates phase gradient rate (PGR) information into the PU framework. The PGR, calculated by stacking differential interferograms over time, reflects the temporal stability of phase gradients and helps suppress noise. By using PGR as a physically-informed constraint, the proposed method assigns zero weight to arcs crossing high-gradient areas, effectively steering the unwrapping path away from unreliable zones and into more stable regions. This improves phase recovery in areas where conventional assumptions of continuity break down. Without relying on external models or training data, this approach enhances both the robustness and adaptability of phase unwrapping, particularly in high-gradient deformation fields and complex terrain.

## METHODS

In the proposed PGR-MCF method, high-coherence pixels (e.g., coherence > 0.4) are first selected to construct a Delaunay triangulation network, providing the initial spatial structure for unwrapping. Then, phase gradients of each interferogram are calculated using the central difference method, and stacked across time series to obtain a more stable and accurate PGR.

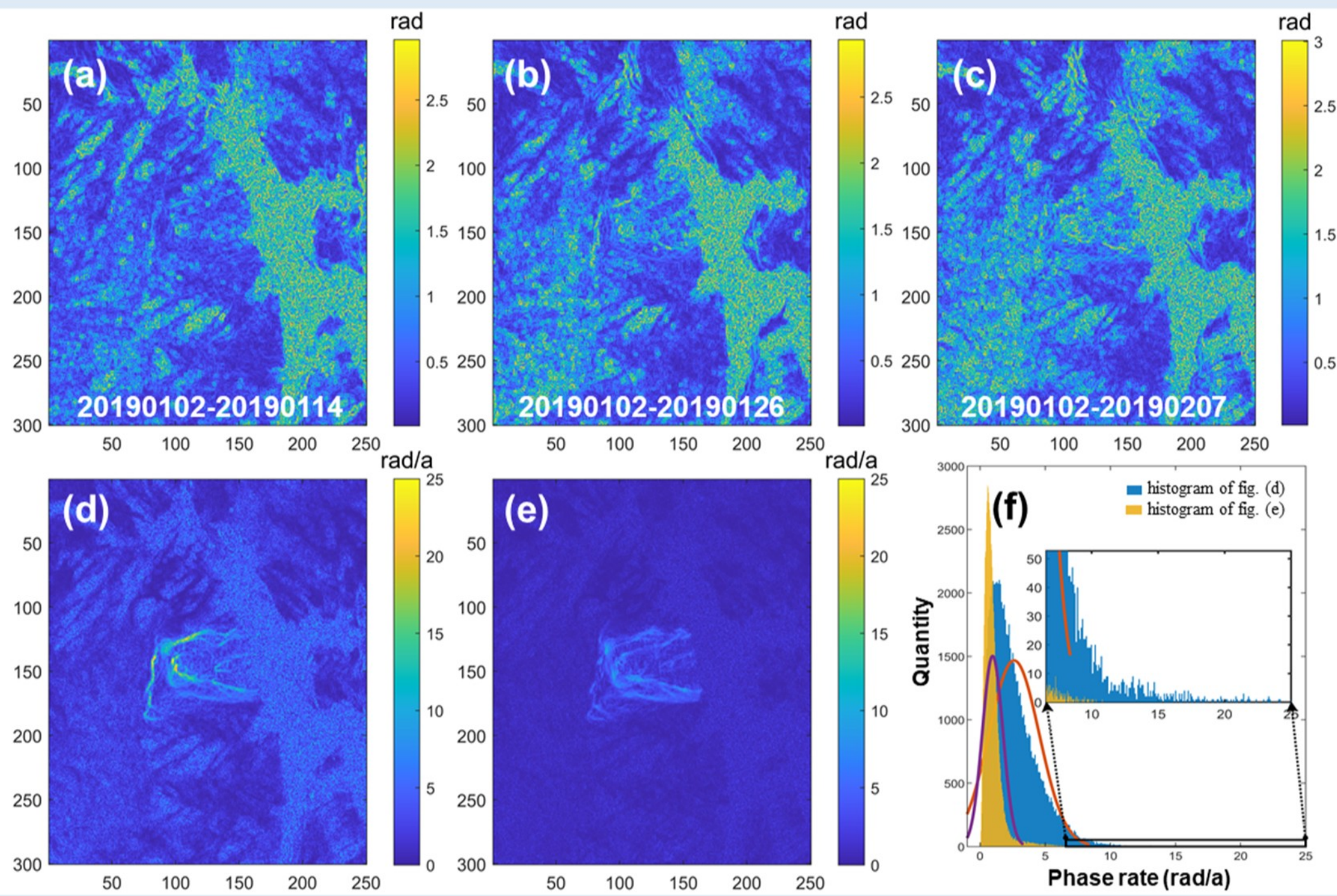


Fig.1 Phase gradient and PGR maps

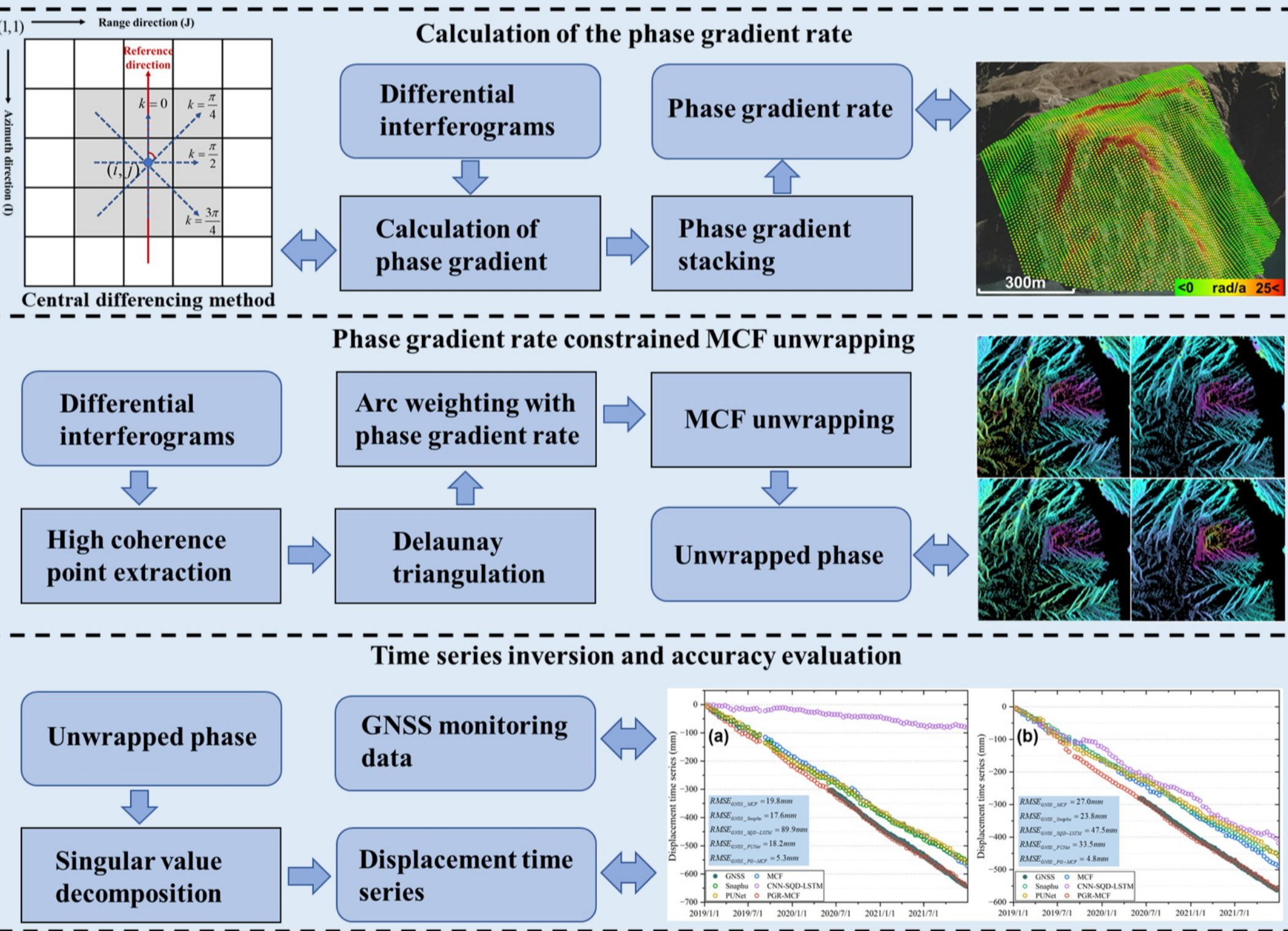
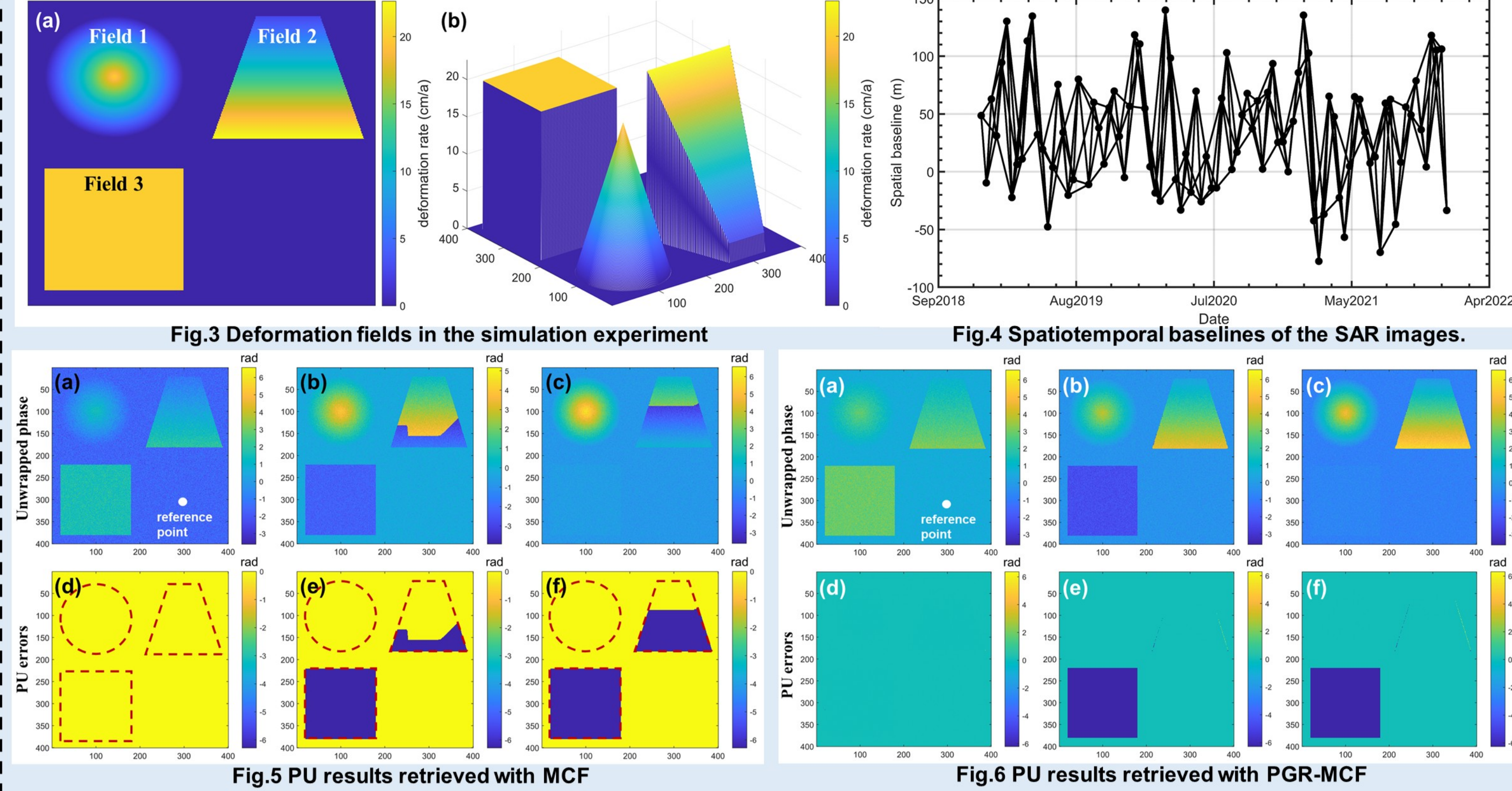


Fig.2 PGR-MCF Technical Flowchart

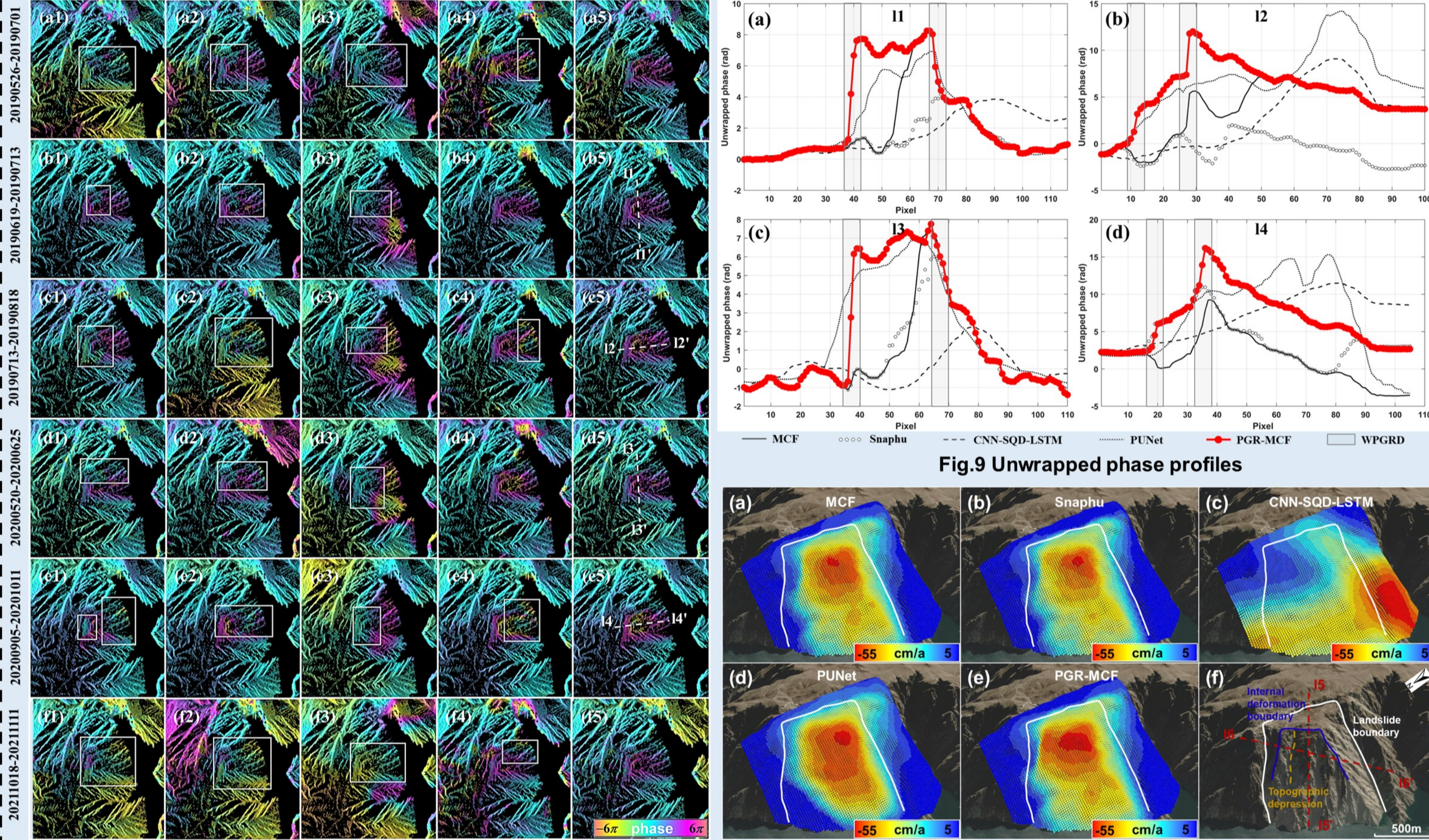
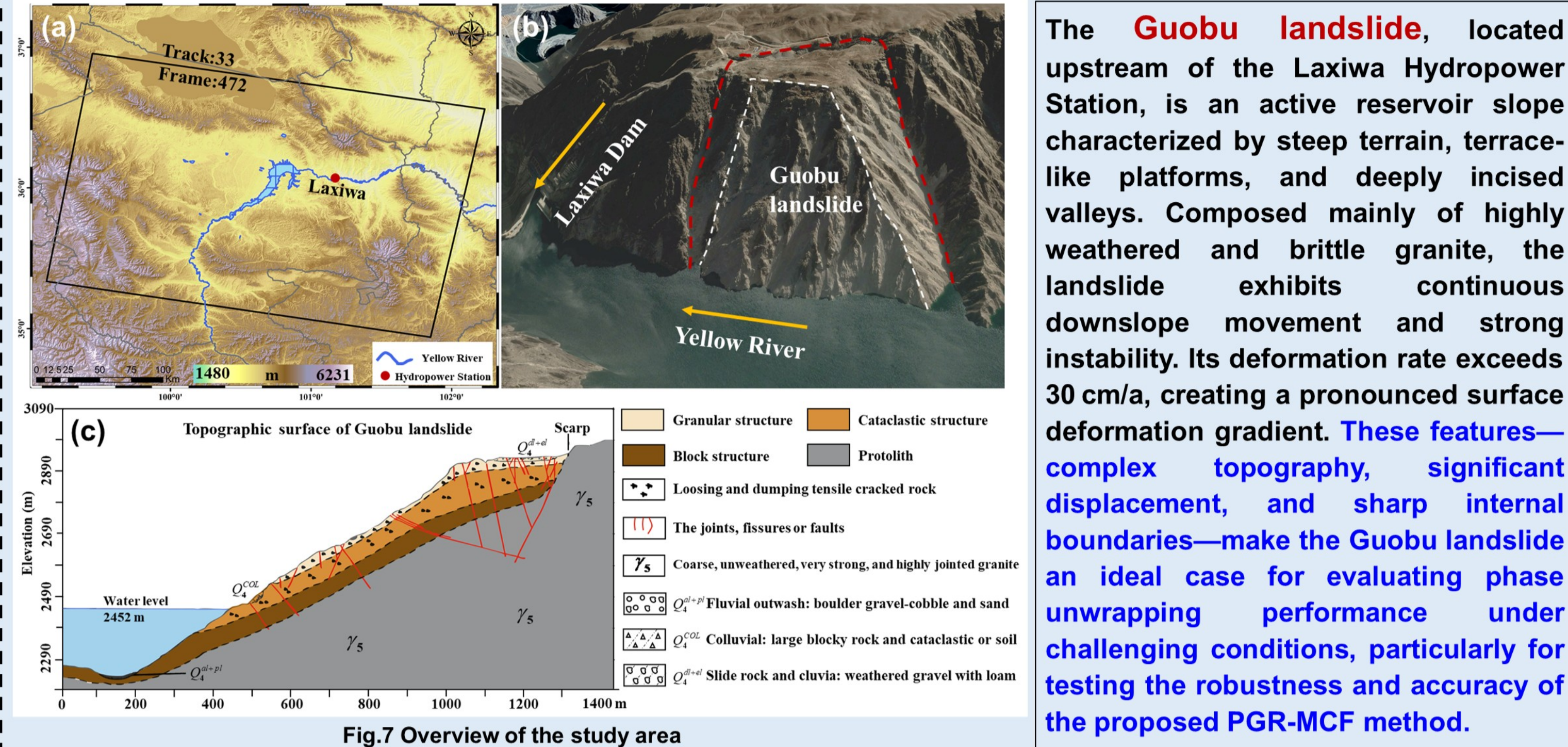
## CONCLUSIONS

This study proposes PGR-MCF, a novel phase unwrapping method that integrates phase gradient rate (PGR) information into the minimum cost flow framework. By constraining the unwrapping path with PGR extracted from stacked differential interferograms, the method avoids high-gradient regions and improves phase recovery accuracy. Simulation results show over 99.5% correct unwrapping in synthetic high-gradient scenarios. In the Guobu landslide case, PGR-MCF reduces the RMSE between InSAR and GNSS deformation time series by over 65% compared to other methods. Fully data-driven and independent of external datasets or prior models, it performs reliably on interferograms with large temporal and spatial baselines. This increases the number of usable interferograms for time series analysis and enhances the robustness of deformation monitoring. Overall, PGR-MCF provides an accurate and adaptable solution for InSAR phase unwrapping in complex deformation environments.

## SIMULATION EXPERIMENT



## REAL DATA EXPERIMENT (GUOBU LANDSLIDE)



After phase unwrapping using different methods, the deformation rates of the Guobu landslide were estimated via Stacking-InSAR (Fig. 10). Positive values indicate motion toward the satellite, and negative values indicate motion away. Among all methods, PGR-MCF yields a more spatially continuous velocity field and captures strong motion (> 50 cm/a) in the landslide center, which is underestimated by MCF and Snaphu. CNN-SQD-LSTM underestimates motion on the left and introduces artifacts on the right, likely due to over-smoothing. While PUNet preserves the general pattern, it loses detail in core and edge areas. Uniquely, PGR-MCF delineates an internal deformation front (blue line), enhancing interpretation of landslide kinematics.

## DISCUSSION

The proposed PGR-MCF method enhances phase unwrapping by integrating stacked PGR to guide paths away from high-gradient regions. This physically-informed constraint reduces unwrapping errors where traditional methods like MCF and deep learning models often fail. By relaxing the reliance on short temporal baselines, PGR-MCF enables reliable unwrapping of long-baseline interferograms, increasing usable observations for time series analysis and improving deformation monitoring accuracy. Notably, it preserves sharp deformation boundaries that are often smoothed or blurred by other methods, allowing clear identification of features such as internal slip surfaces and deformation fronts—crucial for understanding landslides and ground instability. The method is fully data-driven, computationally efficient, and does not depend on external models, offering strong adaptability to various InSAR datasets. These advantages make PGR-MCF a robust and practical tool for deformation monitoring, particularly in complex terrains and rapid hazard assessment.

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